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Technical note:

Performance optimization of irreversible ferromagnetic Stirling heat pumps

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Abstract

An irreversible Stirling heat pump cycle using ferromagnetic materials as a cyclic working substance is established. The influences of finite-rate heat transfer, heat leak, regeneration time, regenerative loss and the internal irreversibility on the cyclic performance of magnetic heat pumps are taken into account. On the basis of the thermodynamic properties of ferromagnetic materials, the performance characteristic of the ferromagnetic Stirling heat pump cycle is investigated. Also, by applying the optimization technology, the relation between the optimal heating load and the coefficient of performance (*COP*) is derived. Furthermore, the maximum heating load and the corresponding *COP* as well as the maximum *COP* and the corresponding heating load are determined and discussed in detail. The results obtained in the present paper may provide some new information for the optimal design and the development of real magnetic heat pumps.

Keywords

Ferromagnetic material; heat pump; optimal performance.

1. Introduction

A magnetic refrigerator, which has more advantages than a gas refrigerator in refrigeration efficiency, reliability, low noise and environmental friendliness, is becoming a promising technology to replace the conventional gas-compression / expansion technique in use today. As some novel room- or near room-temperature magnetic refrigeration materials (magnetic refrigerants) for different temperature ranges are discovered, a number of scholars have focused on their investigations on magnetic refrigeration systems using magnetic materials as a cyclic working substance [1-8]. It is well known that a magnetic refrigerator may be also used as a magnetic heat pump, only their operating aims are different. For the room-temperature region, it is obvious that a magnetic heat pump has its application value. Some scholars paid their attentions to the study of the magnetic heat pump [9-14]. It is valuable and significant work to further explore optimal performances of magnetic heat pumps.

In the present paper, based on the thermodynamic properties of ferromagnetic materials, the optimal performance of irreversible Stirling heat pump using ferromagnetic materials as a cyclic working substance is analyzed and evaluated by employing the optimal thermodynamics approach. The influences of multi-irreversibilities, including finite-rate heat transfer, heat leak, regeneration time, regenerative loss and the irreversibility inside the cyclic working substance, on the optimal performance of magnetic heat pump

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system are revealed. The results obtained are helpful to the optimal design and development of magnetic heat pumps.

2. An irreversible ferromagnetic stirling heat pump system

For homogeneous ferromagnetic materials, according to the phenomenological molecular-field theory, the state function, the fundamental thermodynamic equation and the entropy equation are, respectively, given by [5]

$$M = ng\mu_B JB_J(x) \tag{1}$$

$$du = Tds + \mu_0(H + \lambda M)dM \tag{2}$$

$$s = s_0(T) - \frac{\mu_0 M(H+\lambda M)}{T} + \mu_0 nk \times [\ln\sinh(\frac{2J+1}{2J}x) - \ln\sinh(\frac{1}{2J}x)] \tag{3}$$

Where

$B_J(x) = \frac{2J+1}{2J}\coth(\frac{2J+1}{2J}x) - \frac{1}{2J}\coth(\frac{1}{2J}x)$ is the Brillouin function and $x = g\mu_B J(H+\lambda M)/kT$; n is the number of magnetic moments per unit volume; g , μ_B and J are the Landé factor, the Bohr magneton and the quantum number of the angular momentum, respectively; M and H are magnetization and the magnetic field intensity; $\lambda = 3kT_C/[ng^2\mu_B^2 J(J+1)]$ is the molecular-field constant ; μ_0 is the permeability of vacuum; Tc, T and k are the Curie temperature, absolute temperature and the Boltzmann constant. u and s are, respectively, the internal energy and entropy per unit volume. And s_0 is the entropy of the ferromagnetic system with M = 0 and only a function of temperature T.

On the other hand, a magnetic Stirling heat pump cycle consists of two isothermal processes and two iso-magnetization processes, which can be represented by a M~T diagram, as shown in Fig. 1. In Fig.1, a-b and c-d are two isothermal processes in which the temperatures of cyclic working substance are T_1 and T_2, respectively; b-c and d-a are two iso-magnetization processes in which the magnetizations of cyclic working substance are M_2 and M_1 , respectively. Q_1 and Q_2 are the heats released to the heated space at the temperature T_H and absorbed from the cold reservoir at the temperature T_L by the working substance per cycle, respectively. In addition, Q_{bc} and Q_{da} are the amounts of heat exchange between the working substance and the regenerator during the two iso-magnetization processes, respectively. Moreover, the heat transfer should carry out in finite temperature differences such that the temperatures T_1, T_2 of working substance in the two isothermal processes are different from those of the two heat reservoirs T_H, T_L, namely,

$$T_1 > T_H > T_L > T_2.$$

It is assumed that the heat exchanges between the working substance and the two heat reservoirs obey the Newtonian heat transfer law, i.e.,

$$Q_1 = \alpha(T_1 - T_H)t_1 \tag{4}$$

$$Q_2 = \beta(T_L - T_2)t_2 \tag{5}$$

where t_1 and t_2 are the times of heat exchange between the working substance and the heat reservoirs T_H and T_L, respectively; α and β the corresponding coefficients of heat transfer.

Furthermore, the time, t_3 or t_4, spent on any regenerative process in the Stirling heat pump cycle is proportional to the temperature difference [5], and thus it has

$$t_3 = t_4 = K(T_1 - T_2) \tag{6}$$

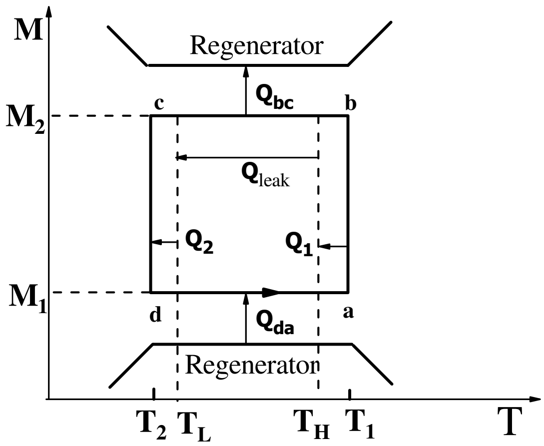


Fig.1: M~T diagram of the irreversible ferromagnetic Stirling heat pump cycle.

Where K is a proportional constant which is independent of temperature.

On the basis of Eq(3), Q_1 can be expressed as :

$$Q_1 = T_1(s_a - s_b) = \mu_0(F + nkY)T_1 \tag{7}$$

Where $F = M_2F_2 - M_1F_1$, $F_1 = (H_a + \lambda M_1)/T_1$, $F_2 = (H_b + \lambda M_2)/T_1$,

$$Y = \ln\left\{\frac{sh\left(\frac{2J+1}{2J}x_a\right)sh\left(\frac{x_b}{2J}\right)}{sh\left(\frac{x_a}{2J}\right)sh\left(\frac{2J+1}{2J}x_b\right)}\right\},$$

$x_a = g\mu_B JF_1/k$, $x_b = g\mu_B JF_2/k$, and H_a and H_b are the magnetic field intensities at the states a and b, respectively. Thus, the cycle period

$$\begin{aligned}\tau &= t_1 + t_2 + t_3 + t_4 \\ &= \mu_0(F + nkY)[\frac{T_1}{\alpha(T_1 - T_H)} + \frac{T_2}{\beta(T_L - T_2)} \\ &+ A(T_1 - T_2)]\end{aligned} \tag{8}$$

Where $A = 2K/(\mu_0(F + nkY))$.

In addition, the heat leak Q_{leak} from the heated space to the cold reservoir also obeys Newtonian heat-transfer law, i.e.,

$$Q_{leak} = \gamma(T_H - T_L)\tau \tag{9}$$

Where γ is the heat leak coefficient. Moreover, when finite-rate heat transfer is taken into account, there is still an additional regenerative loss ΔQ_r which depends on the efficiency η of the regenerator, the heat capacity C_M at constant magnetization and the temperatures, T_1 and T_2, of the cycle working substance in the isothermal processes, i.e.,

$$\Delta Q_r = C_M(1-\eta)(T_1 - T_2) \tag{10}$$

Where $\eta \le 1$. If the efficiency η of the regenerator is equal to 1, it implies that there is not the additional regenerative loss such that $\Delta Q_r = 0$.

Owing to existing eddy currents and other irreversible effects inside the working substance, the ferromagnetic Stirling heat pump cycle is irreversible. According to the second law, we have:

$$\oint (dQ/T) = Q_2/T_2 - Q_1/T_1 < 0 \tag{11}$$

In order to describe quantitatively the effect of the internal dissipations of working substance on the performance of ferromagnetic Stirling heat pump, we introduce an internal irreversibility parameter

$$I = (Q_1/T_1)/(Q_2/T_2) \tag{12}$$

To describe summarily the degree of internal irreversibility and $I \ge 1$. For this reason, Eq. (11) can be rewritten as:

$$I\,Q_2/T_2 - Q_1/T_1 = 0 \tag{13}$$

Eq. (12) shows clearly that when I >1, the cycle is internally irreversible one.

When the irreversibilities mentioned above are all taken into account, the net amount of heats released to the heated space and absorbed from the cold reservoir are given by:

$$Q_H = Q_1 - Q_{leak} - \Delta Q_r \tag{14}$$

$$Q_L = Q_2 - Q_{leak} - \Delta Q_r \tag{15}$$

And, the work input, heating load and *COP* as, respectively,

$$W = Q_H - Q_L = Q_1 - Q_2 \tag{16}$$

$$\Pi = Q_H/\tau \tag{17}$$

$$COP = Q_H/W \tag{18}$$

3. Optimization On The Performance Parameters

It is well known that *COP* and heating load are two important performance parameters of heat pump. By using the above equations, we can derive the mathematical expressions of the heating load and *COP* of the ferromagnetic Stirling heat pump as follows:

$$\begin{aligned}\pi &= [aT_2 + (1-a)T_1]\cdot[\frac{T_1}{\alpha(T_1 - T_H)} \\ &+ \frac{T_2}{\beta(T_L - T_2)} + A(T_1 - T_2)]^{-1} - D\end{aligned} \tag{19}$$

$$COP = \frac{1}{T_1 - T_2/I}\left\{\begin{aligned}&T_1 - a(T_1 - T_2) \\ &-D[\frac{T_1}{\alpha(T_1 - T_H)} + \frac{T_2}{\beta(T_L - T_2)} \\ &+A(T_1 - T_2)]\end{aligned}\right\} \tag{20}$$

Where $a = C_M(1-\eta)/[\mu_0(F+nkY)]$ and $D = \gamma(T_H - T_L)$, and a, D and A are the three parameters which depend on the efficiency η of regenerator, the heat leak coefficient γ and the proportional constant K related to the regenerative time.

For the convenience of calculation, letting $z = T_1$, $y=T_2/T_1$, then Eqs. (19) and (20) may be written as:

$$\pi = [ay+1-a][\frac{1}{\alpha(z-T_H)} + \frac{y}{\beta(T_L - zy)} + A(1-y]^{-1} - D \tag{21}$$

$$COP = \frac{1}{1-y/I}\left\{\begin{array}{l} 1-a+ay-D[\frac{1}{\alpha(z-T_H)}+ \\ \frac{y}{\beta(T_L-zy)} + A(1-y)] \end{array}\right\} \tag{22}$$

Taking the derivatives of Π and COP with respect to z and setting them to equal to zero, that is $\partial\pi/\partial z = 0$, $\partial(COP)/\partial z = 0$, this yields one and the same equation, i.e.,

$$z = \frac{yT_H + \sqrt{\beta/\alpha}T_L}{(1+\sqrt{\beta/\alpha})y} \tag{23}$$

Furthermore, substituting Eq.(23) into Eqs.(21) and (22), we have:

$$\Pi = [ay+1-a][\frac{y}{\varepsilon(T_L - yT_H)} + A(1-y)]^{-1} - D \tag{24}$$

$$COP = (1-y/I)^{-1}\left\{\begin{array}{l} 1-a+ay-D[\frac{y}{\varepsilon(T_L - yT_H)} \\ +A(1-y)] \end{array}\right\} \tag{25}$$

Where $\varepsilon = \alpha\beta/(\sqrt{\alpha}+\sqrt{\beta})^2$. Eqs.(24) and (25) are the two important optimal equations with respect to the heating load and COP and we may discuss the optimal performance characteristics of the irreversible ferromagnetic Stirling heat pump from them.

4. Discussion

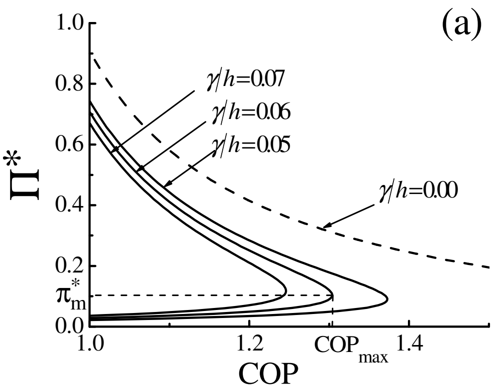
Letting $\alpha = \beta = h$, $T_H/T_L = 1.5$, $a = 0.2$, $AhT_L = 0.01$, $I = 1.02$, one can generate the dimensionless heating load $\Pi*$ versus COP curves, as shown in Fig.2(a), where $\Pi^* = \Pi/hT_L$. From Fig.2 (a) we can see that the influence of heat leak on Π^* ~ COP characteristic. As the value of γ/h decreases, the maximum COP increases significantly and the corresponding dimensionless heating load Π^*_m decreases slightly.

As an example, when $T_H/T_L = 1.5$, $a = 0.2$, $I = 1.02$ are given, one can find that when γ/h =0.05, 0.06, 0.07, COP_{max}=1.371, 1.304, 1.246, and the corresponding dimensionless heating load Π^*_m = 0.092, 0.106, 0.118, respectively. Especially, when the effect of heat leak may be neglected (γ→0), the COP decreases monotonously as the Π^* increases and there does not exist any maximum.

Furthermore, Figs.2(b), 2(c) and 2(d) show, respectively, the effects of the regenerative time parameter AhT_L, the regenerative efficiency parameter a and the internal irreversibility parameter I on the optimal performances of the irreversible ferromagnetic Stirling heat pump. And the value of the related parameters in Figs. 2(b), 2(c) and 2(d) are the same as those used in Fig.2 (a).

Figs. 2(a),(b),(c) and (d) also show that the optimal operating regions of the irreversible ferromagnetic Stirling heat pump should be situated in the parts of the curves with a negative slope, where the COP increases as the heating load decreases and vice versa.

Thus, the optimal COP and dimensionless heating load of the heat pump should satisfy the following relation: $\Pi^* \geq \Pi^*_m$.



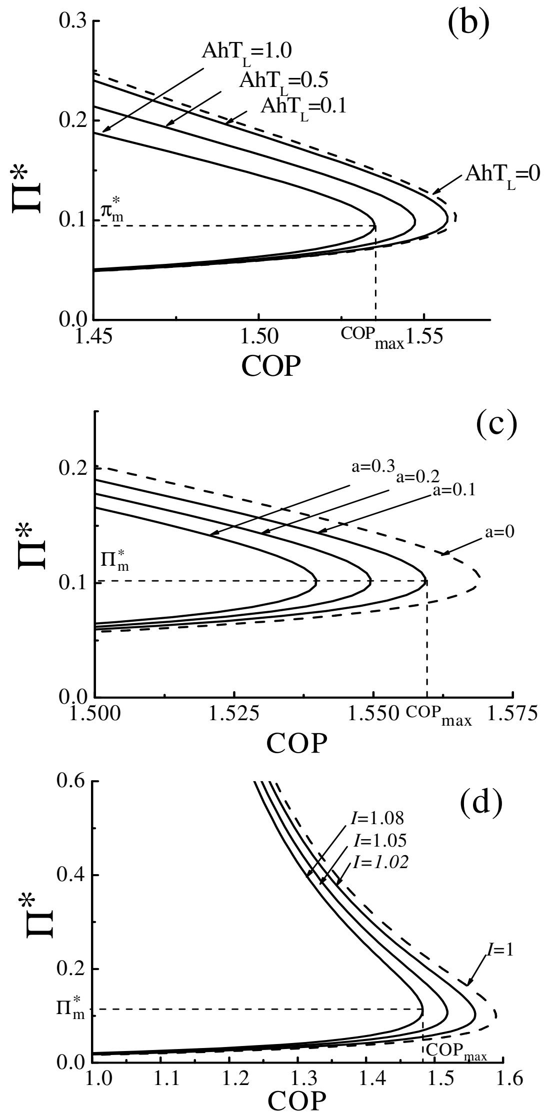


Fig.2: The influences of γ/h (a); AhT_L (b); *a* (c); *I* (d) on the dimensionless heating load Π^* versus the *COP* curves

Fig.3 shows the *COP*~Π^* curves with different temperature ratio T_H/T_L, and in Fig.3 $AhT_L = 0.01$, $a = 0.2$, $\gamma/h = 0.05$, $I = 1.02$ are given. Fig.3 indicates clearly that as the temperature ratio T_H/T_L increases, both the *COP* and the heating load decrease significantly.

It should be pointed out that the maximum *COP* and the corresponding dimensionless heating load Π_m^* are two important performance bounds of the irreversible ferromagnetic Stirling heat pump. The former can determine the upper bound of *COP* and the latter is the lowest value of the heating load of the heat pump. Their variations with the main irreversible parameters, e.g., the heat leak parameter γ, the regenerative time parameter AhT_L, the regenerative efficiency parameter *a* and the internal irreversibility parameter *I* , can be determined further from Eqs.(24) and (25) or Fig.2. In fact, we can also see from Fig.4 that the maximum *COP* decreases significantly and the corresponding dimensionless heating load Π_m^* increases almost linearly as the heat leak parameter increases.

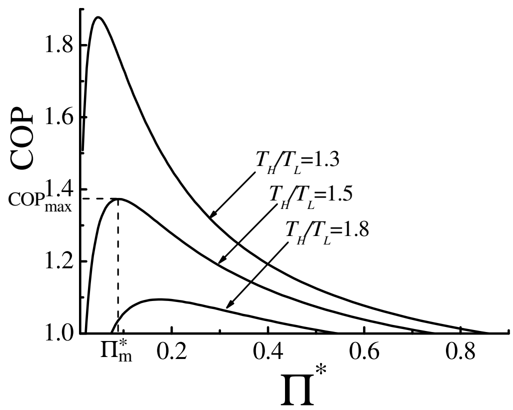


Fig.3: Influence of T_H/T_L on the *COP* versus the dimensionless heating load Π^* curves

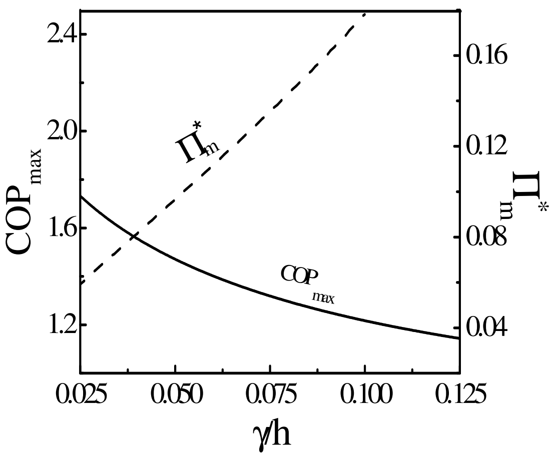


Fig.4: The maximum *COP* and the corresponding dimensionless heating load Π_m^* versus the heat leak parameter γ/h curves

5. Conclusions

The new model of irreversible Stirling heat pump cycle using ferromagnetic materials as a cyclic working substance is established in the present paper, in which the effects of finite rate heat transfer, heat leak between the two heat reservoirs, irreversibility inside the cyclic working substance, regenerative loss and regenerative time on the performance characteristic of the ferromagnetic Stirling heat pump cycle are revealed. Based on the thermodynamic properties of ferromagnetic materials and by using the optimization technology, the mathematical expressions of the heating load and *COP* of the ferromagnetic Stirling heat pump cycle are derived and the optimal performance of the heat pump cycle is analyzed and discussed. Moreover, some important performance bounds including the maximum *COP* and the corresponding heating load are determined and evaluated. The results obtained here may provide some parameter design reference for actual magnetic heat pumps.

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7. Nomenclature

A	parameter related to regenerative time
B_J	Brillouin function
C_M	heat capacitance at iso-magnetization
D	parameter related to heat leak
H	magnetic field intensity
I	internal irreversibility parameter
J	quantum number of angular momentum
M	magnetization
Q	heat
Q_{leak}	heat leak
ΔQ_r	regenerative loss
T	absolute temperature
Tc	Curie temperature
W	work input
COP	coefficient of performance
Π	heating load
Π^*_m	dimensionless heating load at maximum *COP*
a	parameter related to efficiency of regenerator
g	Landé factor
n	number of magnetic moments per unit volume
s	entropy per unit volume
s_0	entropy with M=0
u	internal energy per unit volume
α, β	heat-transfer coefficient
γ	heat leak coefficient
λ	molecular-field constant
η	efficiency of regenerator
μ_B	Bohr magneton
μ_0	permeability of vacuum

References

[1] A. Kitanovski, et al. Thermodynamics of magnetic refrigeration, Int. J. Refrigeration, 2006, 26: 3–21.

[2] B.F. Yu, Q. Gao, B. Zhang, et al. Review on research of room temperature magnetic refrigeration, Int J Refrigeration, 2003, 26: 1–15.

[3] V.K. Pecharsky, K.A. Gschneidner Jr. Magnetocaloric effect and magnetic refrigeration, J MMM, 1999, 200: 44–56.

[4] F. Wu, L.G. Chen. Optimization of irreversible magnetic Stirling cryocoolers. Int J Engineering Science, 2001, 39: 361-368.

[5] G. Lin, O. Tegus, L. Zhang, et al . General performance characteristics of an irreversible ferromagnetic Stirling refrigeration cycle. Physica B, 2004, 344: 147-156.

[6] Z.R. Xia, X.M. Ye, G.X. Lin, et al. Optimization of the performance characteristics in an irreversible magnetic Ericsson refrigeration cycle. Physica B, 2006, 381: 246-255.

[7] X.M. Ye, L.M. Wu, G.X. Lin, et al. Performance optimization of an irreversible ferromagnetic Stirling refrigerator with finite thermal sources, Proc. ECOS '2006, Vol.3: 1303-1310.

[8] X.M. Ye, G.X. Lin, J.C. Chen. Parametric optimization of an irreversible magnetic Ericsson refrigerator with finite heat reservoirs. Physica B, 2007, 391 : 350-356.

[9] D.V. Brown. Magnetic heat pumping near room temperature. J. Appl. Phys. 1976, 47(8): 3673.

[10] J.R. Hull, K.L. Uherka. Magnetic heat pumps for near-room-temperature applications. Energy, 1989, 14(4):177-185.

[11] M.P. Annaorazov, M. Unal, S.A. Nikitin, et al. Magnetocaloric heat-pump cycles based on the AF-F Transition in Fe-Rh Alloys. J. MMM, 2002, 251: 61-73.

[12] B. F. Yu, M. Liu , W. Peter, et al. A review of magnetic refrigerator and heat pump prototypes built before the year 2010. Int. J. Refrigeration, 2010, 33:1029-1060

[13] E. Brück, O. Tegus, D. Thanh, et al. Magnetocaloric refrigeration near room temperature (invited). J M M M , 2007, 310: 2793-2799

[14] K. A. Gschneidner Jr, V. K. Pecharsky, A. O. Tsokol. Recent developments in magnetocaloric materials. Rep. Prog. Phys., 2005, 68: 1479–1539

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